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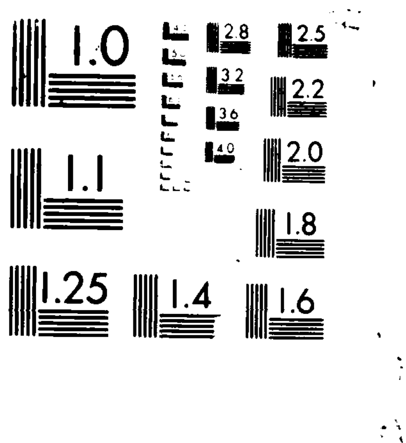
IN BALANCED MODEL (U) PITTSBURGH UNIV PA CENTER FOR
MULTIVARIATE ANALYSIS T MATHEW ET AL NOV 86 TR-86-34
UNCLASSIFIED AFOSR-TR-87-0248 F49620-85-C-0008 F/G 12/1 NL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November 1986

Technical Report No. 86-34

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1 Research supported by the Air Force Office of Scientific Research (AFOSC) under Contract F49620-85-C-0008. The United States Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation hereon.

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ABSTRACT

For any ANOVA model with balanced data involving both fixed and random effects, UMPU and UMPI tests are derived for the significance of a fixed effect or a variance component, under the assumption of normality of random effects. The tests coincide with the usual F-tests. Robustness of the UMPI test against suitable deviations from normality is established.

AMS 1980 Subject Classification: Primary 62F03.
Secondary 62J10, 62F35.

Key words and phrases: Balanced models, fixed effects, random effects, variance components, UMPI, UMPU, elliptically symmetric distributions.

[1]Research supported by Air Force Office of Scientific Research (AFOSR) under contract F49620-85-C-0008.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 87-0248	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Optimum tests for fixed effects and variance components in balanced models		5. TYPE OF REPORT & PERIOD COVERED November 1986
		6. PERFORMING ORG. REPORT NUMBER 86-34
7. AUTHOR(s) Thomas Mathew and Bimal Kumar Sinha		8. CONTRACT OR GRANT NUMBER(s) F49620-85-C-0008
9. PERFORMING ORGANIZATION NAME AND ADDRESS Center for Multivariate Analysis, Fifth Floor Thackeray Hall, University of Pittsburgh, Pittsburgh, PA 15260		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304 A5
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research, Department of the Air Force, Bolling Air Force Base, DC 20332 NM		12. REPORT DATE November 1986
		13. NUMBER OF PAGES 11
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Same as 11.		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19

Key words and phrases: Balanced models, fixed effects, random effects, variance components, UMPI, UMPU, elliptically symmetric distributions.

20 ABSTRACT (Continue on reverse side if necessary and identify by block number)

For any ANOVA model with balanced data involving both fixed and random effects, UMPU and UMPI tests are derived for the significance of a fixed effect or a variance component, under the assumption of normality of random effects. The tests coincide with the usual F-tests. Robustness of the UMPI test against suitable deviations from normality is established.

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

16. Spjøvoll, E. (1967). Optimum invariant tests in unbalanced variance components models. Ann. Math. Statist. 38, 422-429.

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1. <u>Introduction</u>. Inference problems in variance components models have been investigated extensively in the literature. These include estimation of fixed effects and variance components (the latter mostly by the MINQUE method and its modifications) as well as tests of hypotheses for both fixed effects and variance components in general mixed ANOVA models. For such models with balanced data, it is known that the usual tests for fixed effects are optimal (UMPU, UMPI) [vide Seifert (1978, 1979)]. However, for tests on variance components, optimal tests are known only for special models like the one way classificaiton model and two way classification model with or without interaction [vide Herbach (1959), Spjøtvoll (1967), Das and Sinha (1986)]. Some exact tests are obtained in Seifert (1981, 1985). It may be mentioned that the recent book by Arnold (1981) while dealing with tests of variance components mentions no optimum tests but only some valid exact tests.

In this paper a general balanced ANOVA model with mixed effects is considered and UMPU and UMPI tests are obtained for hypotheses on fixed effects as well as variance components. The tests are derived under the usual assumption of normality of the random effects and it is shown that the tests coincide with the standard F-tests. Null, nonnull and optimality robustness of the UMPI test [vide Kariya and Sinha (1985)] against suitable deviations from normality of random effects is established. It may be mentioned that for unbalanced mixed effects models, even though exact tests are available in some cases [vide Thompson (1955a, 1955b), Thomsen (1975), Pincus (1977) and Seely and El-Bassiouni (1983)], the problem of deriving optimum tests for variance components in general is still open (see, however, Das and Sinha (1986) and Spjøtvoll (1967) for the one way unbalanced random effects model). This is currently under investigation and will be reported elsewhere.

2. <u>Mixed Models with Balanced Data and a Canonical Form.</u>

The model under consideration is

(2.1) $$Y = X_1\alpha_1 + \ldots + X_k\alpha_k + Z_1u_1 + \ldots + Z_cu_c.$$

Here Y is the n-dimensional vector of observations, $X_1 = 1_n$ (the n component vector of ones), $\alpha_1 = \mu$, the general effect, α_i's are the vectors of fixed effects ($i = 2,\ldots k$), u_j's are the vectors of random effects ($j = 1,2,\ldots c-1$) due to the various factors (crossed or nested) and their interactions. We assume that $Z_c = I_n$, the identity matrix of order n, and u_c is the vector of experimental errors. The u_j's are assumed to be independent random variables distributed as normal with means 0 and covariance matrices $\sigma_j^2 I_{k_n}$ ($j = 1,2,\ldots c$) ($\sigma_j^2 \geq 0$ for $j = 1,2,\ldots c-1$ and $\sigma_c^2 > 0$). Thus $E(Y) = \sum_{i=1} X_i\alpha_i = X\alpha$, where $X = (X_1:\ldots:X_k)$ and $\alpha = (\alpha_1', \alpha_2', \ldots, \alpha_k')'$ and $D(Y) = V = \sum_{j=1}^{c} \sigma_j^2 V_j$, where $V_j = Z_j Z_j'$ ($j = 1,2,\ldots,c$) and $V_c = I_n$. Each X_i (and each Z_j) is a kronecker product of identity matrices and the vectors 1 of appropriate orders. Hence V_j is a kronecker product of I and J matrices, where $J = 11'$. For a detailed description of models with balanced data, we refer to Seifert (1979) or Anderson et. al. (1984).

Let P_i ($i = 1,2,\ldots,k$) and Q_j ($j = 1,2,\ldots,c-1$) be projectors where $P_1 = \frac{1}{n} J_n$ such that $Y'P_iY$ and $Y'Q_jY$ are the sum of squares due to α_i and u_j (as in the fixed effects models) respectively. Clearly P_i's and Q_j's satisfy $P_iP_\ell = 0$ ($i \neq \ell$), $Q_jQ_\ell = 0$ ($j \neq \ell$) and $P_iQ_j = 0$ for all i and j. The error sum of squares is $Y'(I_n - \sum_{i=1}^{k} P_i - \sum_{j=1}^{c-1} Q_j)Y = Y'Q_cY$ (say). We note that each P_i (and each Q_j) is a kronecker product of matrices of the form I_a, $\frac{1}{b} J_b$ and $I_d - \frac{1}{d} J_d$ (this follows from the rules for writing down the sum of squares for balanced data given in Searle (1971, pp.389-404); see also Seifert (1979), Section 2). Consequently, for each i and j, V_iP_j (and V_iQ_j) is either zero or a multiple of P_j (respectively Q_j). Since $V = \sum_{i=1}^{c} \sigma_i^2 V_i$, we get $VP_j = \beta_j P_j$ and $VQ_j = \delta_j Q_j$, where β_j and δ_j are positive linear combinations of σ_i^2's. Hence

$$V = \sum_{j=1}^{k} VP_j + \sum_{j=1}^{c} VQ_j = \sum_{j=1}^{k} \beta_j P_j + \sum_{j=1}^{c} \delta_j Q_j. \tag{2.2}$$

Consequently

$$V^{-1} = \sum_{j=1}^{k} \theta_j P_j + \sum_{j=1}^{c} \tau_j Q_j, \text{ where } \theta_j = 1/\beta_j \text{ and } \tau_j = 1/\delta_j. \tag{2.3}$$

If $P = \sum_{j=1}^{k} P_j$, then $X\hat{\beta} = PY$ is the BLUE of $X\beta$. If rank $(P_j) = p_j$ and rank $(Q_j) = q_j$, then $|V^{-1}| = (\prod_{j=1}^{k} \theta_j^{p_j})(\prod_{j=1}^{c} \tau_j^{q_j})$.

Using the above expressions for V^{-1} and $|V|$, and using the relation $(Y - X\beta)'V^{-1}(Y - X\beta) = (Y - PY)'V^{-1}(Y - PY) + (PY - X\beta)'V^{-1}(PY - X\beta) = Y'QV^{-1}QY + (PY - X\beta)'V^{-1}(PY - X\beta)$ (where $I - P = Q = \sum_{i=1}^{c} Q_i$), the density of Y can be written as

$$(2.4) \qquad f(y) = (2\pi)^{-n/2} \left(\prod_{j=1}^{k} \theta_j^{p_j/2} \right) \left(\prod_{j=1}^{c} \tau_j^{q_j/2} \right) \exp\left(-\frac{1}{2}\left[\sum_{j=1}^{c} \tau_j y'Q_j y + \sum_{i=1}^{k} \theta_i (S_i'y - \gamma_i)'(S_i'y - \gamma_i)\right]\right)$$

where we write $P_i = S_i S_i' (S_i'S_i = I_{p_i})$ and $\gamma_i = S_i'X\beta$. The parameter space is $\Omega = \{\tau, \theta, \gamma : \tau_j > 0, \forall j;\ \theta_i > 0, \forall i;\ \gamma_i \in R^{p_i}, \forall i\}$. Clearly $S_i'Y$ $(i = 1,2,\ldots,k)$ and $Y'Q_jY$ $(j = 1,2,\ldots,c)$ jointly form a complete sufficient statistic. From the definition of τ_i and θ_j, it follows that $\frac{1}{q_j}E(Y'Q_jY) = \frac{1}{\tau_j}$ and $\frac{1}{p_j} E[(S_i'Y - \gamma_i)'(S_i'Y - \gamma_i)] = \frac{1}{\theta_j}$. The terms $\tau_j Y'Q_jY$ and $\theta_i(S_i'Y - \gamma_i)'(S_i'Y - \gamma_i)$ are independent and have central chisquared distributions with degrees of freedom q_j and p_i respectively (see Searle (1971), p. 409). The hypothesis of interest on the fixed effects is $H_\gamma : \gamma_i = 0$ for any given i. An exact test can be obtained if θ_i coincides with one of the τ_j's. Thus, if $\theta_i = \tau_1$, then under H_γ, $\frac{Y'P_iY/p_i}{Y'Q_1Y/q_1}$ has central F-distribution. For many balanced models, testing $\sigma_i^2=0$ is equivalent to testing if two τ_j's are equal. For testing $H_\tau : \tau_i = \tau_j$, an F-test can be obtained similar to the test for H_γ described above.

Remark. It is not always true that θ_i will coincide with one of the τ_j's. It is also not true that $\sigma^2_i = 0$ is always equivalent to the equality of two τ_j's. A counter-example for the latter is a balanced model involving 3 factors A, B, C and the two factor interactions AB and AC, where C is nested within B and all the effects are assumed to be random. The expected values of the various sum of squares are given in Table 9.4 in Searle (1971), p. 394 (also given on p. 411). It can be verified that $\sigma^2_b = 0$ (in Searle's notation) is not equivalent to the equality of two τ_j's in our notation.

3. Optimal Tests and Their Robustness.

Writing $v_j = Y'Q_jY$ $(j = 1,2,\ldots,c)$ and $w_i = S_i'Y$ $(i = 1,2,\ldots,k)$, it follows from (2.4) that the joint density of $v_1,\ldots,v_c$, $w_1,\ldots,w_k$ is given by

$$(3.1) \qquad c(\tau,\theta) \prod_{j=1}^{c} v_j^{\frac{q_j}{2}-1} \exp(-\tfrac{1}{2}[\sum_{j=1}^{c} \tau_j v_j + \sum_{i=1}^{k} \theta_i (w_i - \gamma_i)'(w_i - \gamma_i)]).$$

a) To test $H_\gamma : \gamma_k = 0$ vs $H_1 : \gamma_k \neq 0$. Assume $\theta_k = \tau_1$, so that an exact test is the F-test based on $w_k'w_k/v_1$. If γ_k is a scalar, then it follows from standard results of multiparameter exponential family that this test is UMPU (see Lehmann (1959), Chapter 4). However, if the dimension of γ_k is more than one, then a UMPU test does not exist. In this case a UMPI test (which coincides with the above F-test) can be derived easily. For this, we note that the above testing problem is invariant under the group of transformations $(v_1,\ldots v_c, w_1,\ldots,w_k) \to \alpha(v_1,\ldots,v_c, w_1 + \mu_1,\ldots,w_{k-1} + \mu_{k-1}, Pw_k)$, where $\alpha > 0$, μ_i's are arbitrary vectors and P is an orthogonal matrix. A maximal invariant with respect to the above group is given by $(\frac{w_k'w_k}{v_1}, \frac{v_2}{v_1},\ldots\frac{v_c}{v_1}) = (T_1, T_2,\ldots T_c) = T'$ (say). The null distribution of T can be computed as

$$(3.2) \qquad \text{Constant} \times \prod_{i=2}^{c} \xi_i^{q_i/2} h(T) \left[1 + \xi_2 \frac{T_2}{1+T_1} + \ldots + \xi_c \frac{T_c}{1+T_1}\right]^{-\frac{1}{2}[p_k + \sum_{i=1}^{c} q_i]},$$

where $\xi_i = \frac{\tau_i}{\tau_1}$ and $h(T)$ is a function of T. Thus, under H_0, $Z = (T_2/1 + T_1,\ldots,T_c/1 + T_1)$ is sufficient for the nuisance parameters $(\xi_2,\ldots,\xi_c)$ and it can be shown that the distribution of Z is complete. On the other hand, the nonnull distribution of T is given by

$$(3.3) \qquad \prod_{i=2}^{c} \xi_i^{q_i/2} h(T) \sum_{r=0}^{\infty} \left(\frac{\gamma_k'\gamma_k}{\tau_1}\right)^r d_r (T_1/1 + T_1)^r \left[1 + \xi_2 \frac{T_2}{1+T_1} + \ldots + \xi_c \frac{T_c}{1+T_1}\right]^{-\frac{1}{2}[p_k + \sum_{k=1}^{c} q_i] - r},$$

where $d_\gamma > 0$ are constants. Given Z, this is monotone in $T_1/1 + T_1$. Since the null distribution of $T_1/1 + T_1$ is independent of any parameters, it follows from Basu's theorem (Lehmann (1959), page 162) that $T_1/1 + T_1$ and Z are distributed independently, under H_0. Consequently, the test based on $T_1/1+ T_1$, i.e., the F-test based on T_1, is UMPI.

b. <u>To test H_τ: $\tau_1 = \tau_2$ vs H_1: $\tau_2 > \tau_1$.</u> In this case the F-test based on v_2/v_1 is clearly UMPU. To show that the same test is also UMPI, we note that the above testing problem is invariant under the group of transformations $(v_1,\ldots,v_c,w_1,\ldots,w_k) \to (\alpha v_1,\ldots,\alpha v_c, \sqrt{\alpha}\,(w_1 + \mu_1), \ldots \sqrt{\alpha}\,(w_k + \mu_k))$, where $\alpha > 0$ and μ_i's are arbitrary vectors. A maximal invariant is $(\frac{v_2}{v_1}, \ldots, \frac{v_c}{v_1}) = T^*$ (say). From (3.1), the density of T^* is obtained as

$$(3.4)\quad f(T^*|\xi) = c(\xi)[1 + \xi_2 T_2 + \ldots + \xi_c T_c]^{-q/2} h(T^*)$$

where $q = \sum_{j=1}^{c} q_j$ and $\xi_j = \tau_j/\tau_1$, $j = 2,\ldots,c$. Under the null hypothesis H_τ: $\xi_2 = 1$, $(\frac{T_3}{1+T_2}, \ldots, \frac{T_c}{1+T_2})$ is sufficient for the nuisance parameters $(\xi_3,\ldots,\xi_c)$ and it can be shown that its null distribution is complete. Arguing as before, it can be seen that the test based on v_2/v_1 is UMPI.

We shall now discuss briefly the robustness of the above tests when Y has an elliptically symmetric distribution. Thus the density of Y is

$$(3.4)\qquad f(y) = |V|^{-\frac{1}{2}} \phi[(y - X\beta)' V^{-1}(y - X\beta)]$$

where ϕ is a nonnegative function on $(0,\infty)$ satisfying $\int_{R^n} \phi(u'u)du = 1$. Write $Q_j = S_j^* S_j^{*'}$ and $W_j^* = S_j^{*'} Y$ where $S_j^{*'} S_j^* = I_{q_j}$, $j = 1,\ldots,c$. Making the orthogonal transformation: $Y \to W = (W_1',\ldots,W_k',W_1^{*'},\ldots,W_c^{*'})'$, it follows that the density of W is

$$(3.5)\quad f(w) = \Big(\prod_{j=1}^{k} \theta_j^{p_j/2}\Big)\Big(\prod_{j=1}^{c} \tau_j^{q_j/2}\Big)\phi\Big[\sum_{j=1}^{c} \tau_j w_j^{*'} w_j^* + \sum_{i=1}^{k} \theta_i (w_i - \gamma_i)'(w_i - \gamma_i)\Big]$$

Since ϕ is arbitrary, it is clear that the standard argument of multiparameter exponential family to claim UMPU properties of the appropriate F-tests for the hypotheses $H_\gamma:\gamma_k = 0$ and $H_\tau:\tau_1 = \tau_2$ breaks down. However, the principal of invariance still applies and the following results hold.

(a)' For testing $H_\gamma:\gamma_k = 0$ versus $H_1:\gamma_k \neq 0$, assuming $\theta_k = \tau_1$, the F-test based on $w_k'w_k/w_1^{*\prime}w_1^*$, which is the same as $w_k'w_k/v_1$, is UMPI whenever ϕ is convex. This test is also null robust. This follows essentially from Kariya (1981).

(b)' For testing $H_\tau:\tau_1 = \tau_2$ versus $H_1:\tau_2 > \tau_1$, as already observed, the problem is invariant under the group G of transformations $\{g:g = (\alpha, \mu_1,\ldots,\mu_k), \alpha > 0, \mu_i$'s are arbitrary vectors$\}$ acting on w as $g(w) = \alpha((w_1 + \mu_1)',\ldots,(w_k + \mu_k)',w_1^{*\prime},\ldots,w_c^{*\prime})'$. Then $d\mu_1,\ldots,d\mu_k$ $d\alpha/\alpha$ is a left invariant measure on G, then applying the representation theorem due to Wijsman (1967), the ratio of the nonnull to null distributions of T is given by

$$(3.6)\qquad R = \frac{\int_G f(g(w)|H_1)J^{-1}d\mu_1\ldots d\mu_k d\alpha/\alpha}{\int_G f(g(w)|H_\tau)J^{-1}d\mu_1\ldots d\mu_k d\alpha/\alpha}$$

where J is the jacobian of the transformation $w \to g(w)$. Using (3.5) and the result of Dawid (1977), the numerator of R simplifies to

$$(3.7)\qquad \Big(\prod_{j=1}^{k}\theta_j^{p_j/2}\Big)\Big(\prod_{j=1}^{c}\tau_j^{q_j/2}\Big)\int_0^\infty\int_{R^{p_1}}\int_{R^{p_k}}\phi\Big[\alpha^2\sum_{j=1}^{c}\tau_j w_j^{*\prime}w_j^* +$$

$$+\ \alpha^2\sum_{i=1}^{k}\theta_i(w_i + \mu_i - \gamma_i)'(w_i + \mu_i - \gamma_i)\Big]\cdot\alpha^n\cdot d\mu_1\ldots d\mu_k d\alpha/\alpha$$

$$= \Big(\prod_{j=1}^{c}\tau_j^{q_j/2}\Big)\int_0^\infty\tilde\phi\Big[\alpha^2\sum_{j=1}^{c}\tau_j w_j^{*\prime}w_j^*\Big]\alpha^{n-s-1}d\alpha$$

$$= \Big(\prod_{j=1}^{c}\tau_j^{q_j/2}\Big)\Big(\sum_{j=1}^{c}\tau_j w_j^{*\prime}w_j^*\Big)^{-(n-s)/2}\cdot\Big\{\int_0^\infty\tilde\phi(\alpha^2)\alpha^{n-s-1}d\alpha\Big\}$$

where $\tilde\phi(x) = \int_{R^{p_1}}\cdots\int_{R^{p_k}}\phi(x + u_1'u_1 + \ldots + u_k'u_k)du_1\ldots du_k$ and $s = \sum_{j=1}^{k}p_j/2$.

Since the denominator of R corresponds to the expression in (3.7) under H_T, follows that the ratio R is independent of ϕ. This means that the normal theory result obtains. Consequently, we have established the null, nonnull and optimality robustness of the F-test based on v_2/v_1. See Kariya and Sinha (1985) for detail.

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